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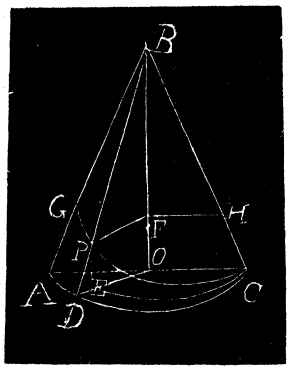
two consecutive points on the curve. Draw PE perpendicular to DO. Then $PC=d\sigma$, $CE=ds$, $PE=dz$, $DE=dr$, $\angle COE=d\theta$, $\angle DPE=\angle DBO=\gamma$.



Then $DE=PD\sin\gamma=PC\sin\gamma\sin\beta$.

$\therefore\ dr=\sin\gamma\sin\beta d\sigma$.

$\therefore\ \sigma=\dfrac{r}{\sin\gamma\sin\beta}=\dfrac{a}{\sin\gamma\sin\beta}$, where $a=r$.

$PE=PD\cos\gamma=PC\cos\gamma\sin\beta$.

$\therefore\ dz=\cos\gamma\sin\beta d\sigma$. $\therefore\ z=\sigma\cos\gamma\sin\beta$.

$CE=\sqrt{(CP^2-PE^2)}$.

$\therefore\ ds=\sqrt{(d\sigma^2-dz^2)}=d\sigma\sqrt{(1-\cos^2\gamma\sin^2\beta)}$.

$\therefore\ s=\sigma\sqrt{(1-\cos^2\gamma\sin^2\beta)}=\dfrac{a\sqrt{(1-\cos^2\gamma\sin^2\beta)}}{\sin\gamma\sin\beta}$

$r.PFH=PH=PC\cos\beta$.

$\therefore\ rd\theta=\cos\beta d\sigma$.

$\therefore\ d\theta=\dfrac{\cos\beta d\sigma}{r}=\dfrac{\cot\beta}{\sin\gamma}.\dfrac{d\sigma}{\sigma}$. $\therefore\ \theta=\dfrac{\cot\beta}{\sin\gamma}\log\sigma$.

$x=r\cos\theta$, $y=r\sin\theta$, $z=\sigma\cos\gamma\sin\beta=r\cot\gamma$ are the equations to the curve. σ and s as given above are the values asked for in the problem.

MECHANICS.

77. Proposed by ELMER SCHUYLER, High Bridge, N. J.

At what elevation must a shell be projected with a velocity of 400 feet that it may range 7500 feet on a plane which descends at an angle of 30°?

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; J. SCHEFFER, A. M., Hagerstown, Md., and the PROPOSER.

Let $OR=7500$ feet$=R$, $V=400$ feet, $\angle POS=e=$angle of elevation, and $\angle ROS=i=30°$.

$\therefore\ PR : OP=\sin(e+i) : \cos i$.

But $OP=vt$, and $PR=\frac{1}{2}gt^2$.

$\therefore\ t=2v\sin(e+i)/g\cos i$. $OR : PR=\cos e : \sin(e+i)$.

$\therefore\ R=gt^2\cos e/2\sin(e+i)=2v^2\sin(e+i)\cos e/g\cos^2 i$.

$$\therefore\ \tan e=\frac{v^2\pm\sqrt{(2v^2gR\sin i-g^2R^2\cos^2 i+v^4)}}{gR\cos i}=\frac{160000\pm 143348.62575}{208928.62775}.$$

$\therefore\ \tan e=1.451925$ or $.079699$.

$\therefore\ e=55°\ 26'\ 36''$ or $4°\ 33'\ 24''$.

II. Solution by J. C. NAGLE, M. A., M. C. E., Professor of Civil Engineering and Physics, Agricultural and Mechanical College, College Station, Texas.

If the point of projection be origin and the path be regarded as a parabola its equation will be

$$y = x\tan\alpha - \frac{gx^2}{2v^2\cos^2\alpha} \ldots (1),$$

where α=angle of elevation.

At the point where the shell is to strike the plane, $y=-3750$ feet and $x=6495$ feet. Inserting these values in equation (1) there results

$$-3750\times 400\cos^2\alpha = 6495\times 200\times \sin\alpha\cos\alpha - (1299)^2.$$

For $\cos^2\alpha$ write $1+\cos 2\alpha$, and for $2\sin\alpha\cos\alpha$ write $\sin 2\alpha$, and as $\sin 2\alpha = \sqrt{(1-\cos^2 2\alpha)}$, we get

$$\sqrt{(1-\cos^2 2\alpha)} = .722 - .577\cos 2\alpha \ldots (2).$$

Solving (2) we get $\cos 2\alpha = 314 \pm .676$.

$\therefore 2\alpha = 8^\circ\ 48'$ or $111^\circ\ 12'$ and $\alpha = 4^\circ\ 24'$ or $55^\circ\ 36'$, which values satisfy the condition

$$\alpha'' - \tfrac{1}{2}(\tfrac{1}{2}\pi - 30) = \tfrac{1}{2}(\tfrac{1}{2}\pi - 30) - \alpha'.$$

See Tait and Steele's *Dynamics of a Particle*, page 90.

Also solved by *P. H. PHILBRICK*.

78. Proposed by ALOIS F. KOVARIK, Professor of Mathematics, Decorah Institute, Decorah, Iowa.

A cone and a cylinder having equal heights and equal circular bases are filled with water; if they have equal holes in the bases, respectively, how many times as long will it take the cylinder to empty as the cone?

I. Solution by P. H. PHILBRICK, C. E., Lake Charles, La.

Let r=radius of base, h=altitude, k=area of orifice, and x=height of the water at the time t.

I. For the cylinder, the discharge in time dt is, $k\sqrt{(2gx)}dt$; and since in the same time the surface of the water descends a distance dx, the quantity in the vessel is *lessened* $\pi r^2 dx$.

$\therefore k\sqrt{(2gx)}dt = \pi r^2 dx$, and

$$t = -\frac{\pi r^2}{k\sqrt{(2g)}}\int \frac{dx}{\sqrt{x}} = -\frac{2\pi r^2}{k\sqrt{(2g)}}x^{\frac{1}{2}} + c = \frac{2\pi r^2}{k\sqrt{(2g)}}(h^{\frac{1}{2}} - x^{\frac{1}{2}}).$$

II. For the cone, we have $y=(r/h)(h-x)$, and the area of section

$$=\pi y^2 = \frac{\pi r^2}{h^2}(h-x)^2.$$